Engineering Notes

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Scaling Analysis of Thermoacoustic Convection in a Zero-Gravity Environment

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Introduction

THERMOACOUSTIC convection (TAC) may be defined as energy transport by acoustic waves that are of thermal origin; that is, by acoustic waves that are induced by rapid addition of thermal energy to a fluid. Such acoustic waves may be produced by the addition of energy as heat transfer at a fluid boundary or within the body of a fluid by processes such as chemical reactions or the absorption of electromagnetic radiation. In any case, the basic mechanism by which the acoustic waves are generated is quite simple: when the internal energy of a given fluid particle is suddenly increased, there is a resulting rapid expansion of the volume occupied by the particle. This expansion produces the pressure waves in the fluid which, in turn, produce a convective motion.

TAC is not only a heat-transfer mechanism of fundamental scientific interest, but a phenomenon that will play an important role in a number of practical applications such as space manufacturing processes, spacecraft fluid handling and storage systems, the stability of combustion systems, and the stability of cryogenic flows. In spite of the many potential applications involving TAC, however, no conclusive experimental data on TAC have been collected to date. Since these applications all involve a low-gravity environment, one of the authors recently began a detailed experimental investigation of thermoacoustic convection using the drop tower facilities at the NASA Lewis Research Center. The design of the experimental apparatus for this investigation required the performance of a scaling analysis of TAC phenomena in a low-gravity environment. This paper presents the results of this analysis. The interpretation of these results provides a rational explanation for discrepancies among previous analytical studies of TAC.

Analysis

A scaling analysis of thermoacoustic convection was performed for the simple one-dimensional configuration shown in Fig. 1. An ideal gas of known properties is assumed to be confined in the region between two infinite parallel plates separated by a distance, L, and located in a zero-gravity environment. The gas is initially at rest and in thermal equilibrium with the plates at temperature $\bar{T}_0$. A step change $(\Delta\bar{T}_0)$ in the temperature of one of the plates is then introduced which causes convective heat transfer between the plates owing to an oscillating fluid motion which is acoustical in nature.

Previous numerical solutions of the governing equations for this flow[1,2] have shown that this convective heat transfer is significant for a large step change in the plate temperature. These solutions, however, are based on ad hoc assumptions and portray conflicting results for the velocity distributions in the gas. The present study determines the correct scaling for the flow on a rational basis and thereby eliminates the need for making any ad hoc assumptions to complete the analysis. In order to assess the relative importance of the various terms in the governing equations for this problem, these equations must first be cast in the correct nondimensional form by applying the well-known techniques of scaling analysis.[3,4] As will be shown, the scaling in this problem is complicated by a multiplicity of time scales.

To begin the scaling analysis, an appropriate set of reference quantities must be deduced. In the limit of "very long times," the transient flow between the plates dies out and Newton's second law gives $\partial\bar{P}/\partial\bar{x}=0$, or

$$\bar{P}=(\text{constant})\equiv P' \tag{1}$$

where $\bar{P}$ is the presssure in the gas and $\bar{x}$ is the spatial coordinate defined in Fig. 1. The energy equation reduces to the conduction term

$$d^2\bar{T}/d\bar{x}^2=0 \tag{2}$$

where $\bar{T}$ is the gas temperature. Upon integration this yields the expected linear steady-state temperature profile

$$\bar{T}=\bar{T}_0+(\Delta\bar{T}_0)(1-\bar{x}/L) \tag{3}$$

The spatially averaged pressure, temperature, and density obtained in this long time limit are denoted by P', T', and ρ', respectively, where

$$T'=\bar{T}_0+\tfrac{1}{2}\Delta\bar{T}_0 \tag{4a}$$

$$\rho'=P'L/R\Delta\bar{T}\ln(1+\Delta\bar{T}_0/\bar{T}_0) \tag{4b}$$

and R is the ideal gas constant. These quantities will now serve as reference quantities for normalizing the pressure, temperature, and density, while L, the distance between the plates, will serve as the obvious length scale in the problem.

Next, the various time scales will be introduced. The most obvious time scale is the characteristic time for a pressure wave to travel across the gap between the plates. This time

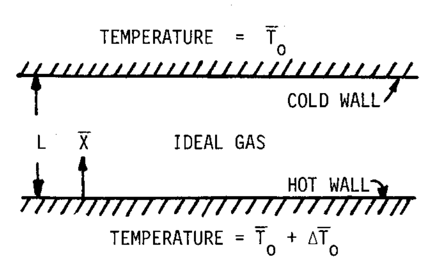

Fig. 1 Geometric configuration to be analyzed.

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scale is given by the ratio of the length scale, L, to the speed of sound in the gas, c, or

$$t_f = L/c = L/\sqrt{\gamma R T'} \tag{5}$$

where γ is the ratio of specific heats of the gas. This is a "fast" time scale with a typical value (e.g., for helium at 273 K and $L=15$ cm) of the order of 10^{-4} s.

A second time scale is given by the characteristic time for the diffusion of thermal energy by conduction, or

$$t_s = L^2/\alpha \tag{6}$$

where α is the thermal diffusivity of the fluid. This is a "slow" time scale with a typical value (e.g., for helium at 273 K and $L=15$ cm) of the order of 4 min. For a general fluid there would be an additional slow time scale given by (L^2/ν), where ν is the kinematic viscosity of the fluid. This is the characteristic time for the diffusion of momentum by viscosity. The present analysis, however, is limited to ideal gases with Prandtl numbers of order unity for which these two slow time scales are of the same order.

The small parameter in the problem, ϵ, is now defined for latter use as

$$\epsilon \equiv t_f/t_s \tag{7}$$

A typical value (e.g., for helium at 273 K, $L=15$ cm) of the small parameter is of the order of 10^{-7}.

There are two velocity scales in the problem. The first is the speed of sound in the gas, c, which is given by

$$c = \sqrt{\gamma R T'} \tag{8}$$

The second velocity scale, taken from linear acoustics, is the characteristic fluid velocity generated by the passage of a pressure wave, or

$$U_{\text{ref}} = \Delta\bar{P}/\rho' c \tag{9}$$

where $\Delta\bar{P} = (\bar{P} - P')$ is the characteristic pressure change. Using the ideal gas equation of state gives

$$U_{\text{ref}} = \Delta\bar{P} c/\gamma P' \tag{10}$$

Normalizing the density, velocity, and spatial coordinate $\bar{x}$ by ρ', U_{ref}, and L, respectively, and the time by an unknown reference time τ, gives the following nondimensional form of the equation for conservation of mass

$$\frac{\partial\rho}{\partial t} + \left(\frac{\tau U_{\text{ref}}}{L}\right)\frac{\partial(\rho u)}{\partial x} = 0 \tag{11}$$

Equation (11) suggests the use of a time scale

$$\tau = L/U_{\text{ref}} = \gamma L P'/c\Delta\bar{P} = (\gamma P'/\Delta\bar{P})t_f \tag{12}$$

so that both terms will be retained in the lowest-order approximation of this equation. Note that τ can be interpreted physically as a "characteristic flow time," since in the limiting case of "very long times" $\tau \to \infty$ because $\Delta\bar{P} = (\bar{P} - P') \to 0$. Thus, when the velocity, temperature, pressure, density, spatial coordinate, and time are normalized by U_{ref}, T', P', ρ', L, and τ, respectively, the nondimensional forms of the governing equations are given by

Conservation of mass:

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{13}$$

Newton's second law:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -\left(\frac{1}{\gamma}\right)\left(\frac{\tau}{t_f}\right)^2\frac{\partial P}{\partial x} + \left(\frac{4}{3}\right)\left(\frac{\tau}{t_s}\right)\frac{\partial^2 u}{\partial x^2} \tag{14}$$

Conservation of energy:

$$\rho\frac{\partial T}{\partial t} + \rho u\frac{\partial T}{\partial x} = \left[\frac{\gamma-1}{\gamma}\right]\left[\frac{\partial P}{\partial t} + \frac{\partial P}{\partial x}\right]$$
$$+\left[\frac{4}{3}\right][\gamma-1][\epsilon]\left[\frac{t_f}{\tau}\right]\left[\frac{\partial u}{\partial x}\right]^2 + \left[\frac{\tau}{t_s}\right]\left[\frac{\partial^2 T}{\partial x^2}\right] \tag{15}$$

State:

$$P = \rho R T \tag{16}$$

In Eqs. (13-16), u, T, P, ρ, x, and t represent the nondimensional velocity, temperature, pressure, density, spatial coordinate, and time, respectively.

Conclusions

A number of important conclusions may now be drawn from a systematic examination of Newton's second law [Eq. (14)] and the energy equation [Eq. (15)] as the time scale, τ, assumes increasingly larger values. First, for "short times," when $\tau = 0(t_f)$, Newton's second law redues to an inertia force-pressure force balance and viscous forces are to lowest order, negligibly small. The energy equation for $\tau = 0(t_f)$ yields a convection-compression work balance with conduction not yet playing an important role. For $\tau = 0(t_s)$ [that is, for moderately long times], the pressure force term completely dominates Newton's second law while the energy equation exhibits a convection-compression work-conduction balance. Finally, for very large times, [for example, when $\tau = 0(t_s/\epsilon)$], Newton's second law is still dominated by the pressure force term; however, to lowest order, only the conduction term remains in the energy equation. These results were anticipated in an earlier discussion of the flow in the limit of "very long times."

Thus, the most important result of this analysis is the observation that the temperature distribution in the fluid is dominated by conduction effects long before any appreciable attenuation of the pressure waves takes place. This explains why the velocity distributions in previously published numerical solutions of this problem[1,2] can disagree by more than an order of magnitude while the temperature distributions are in excellent agreement with each other.

References

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